2/5 Quick Sort and Bound

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Quick Sort

Invented by Tony Hoare

Method

- choose element to partition the array (pivot)
- In linear time move elements less than the pivot to front of Array and elements larger than pivot to back
- Move pivot in between the two groups
- Repeat recursively on front group and back group

Runtime on average case $O(n \log n)$

1) Percentile Proof

Lets consider the sorted array as the concatination of 4 arrays

$$A = [\, A_1 \mid A_2 \mid A_3 \mid A_4 \,]$$

[illegible]

Each array has near equal length

The probability that a pivot chosen from A_2 or A_3 is roughly $1/2$

If a pivot is chosen from A_2 or A_3

Then the run time is in the worst case

$$T(n/4) + T(3n/4) + O(n)$$

~~Since~~ Thus the maximum number of Pivots chosen in the respective $A_2 \cup A_3$ range bc cares

$\log_{4/3}(n)$

The expected number of pivots chosen until an $A_2 \cup A_3$ pivot is chosen is

$$1 \cdot (1/2) + 2 \cdot (1/2) \cdot (1 - 1/2) + 3 (1/2)(1 - 1/2)^2 + \dots$$

which can be expressed as

$$\sum_{i=1}^{\infty} i \, (1/2)^i$$

This is bounded by 2.

Thus the expected number of linear

Thus the expected number of linear search before a desirable pivot is chosen is ≤ 2

So the total expected run time becomes no worse than

$$2n \cdot \log_{4/3}(n) \in O(n \log(n))$$

2) Recurrance Proof (let $E(T(i)) = S_i$)

$$E(T(n)) = \frac{1}{n}\Big((S(0) + S(n-1) + S(1) + S(n-2) + \cdots + S(n-1) + S(0)\Big) + (n-1)$$

Note each term $S(i)$ $i \in [0, n-1]$ occurs twice

$$S(n) = \frac{2}{n} \sum_{i=0}^{n-1} S(i) + n - 1$$

$$S(n-1) = \frac{2}{n-1} \sum_{i=0}^{n-2} S(i) + n - 2$$

$$\left(\frac{n-1}{n}\right) \cdot S(n-1) = \frac{2}{n} \sum_{i=0}^{n-2} S(i) + \frac{(n-2)(n-1)}{n}$$

$$S(n) - \left(\frac{n-1}{n}\right) S(n-1) = \frac{2}{n} S(n-1) + \frac{(n-1)n}{n} - \frac{(n-2)(n-1)}{n}$$

$$S(n) = \frac{n+1}{n} S(n-1) + \frac{2}{n}$$

$$S(n-1) = \frac{n}{n-1} S(n-2) + \frac{2}{n-1}$$

$$S(n) = \frac{n+1}{n-1} S(n-2) + \frac{2}{n} + \frac{2(n+1)}{(n)(n-1)}$$

$$S(n) = \frac{n+1}{n-2} S(n-3) + \frac{2}{n} + \frac{2(n+1)}{n(n-1)} + \frac{2(n+1)}{n(n-2)}$$

$$\leq \frac{n+1}{k} (S(k-1) + \frac{(n+1)}{n} \sum_{i=k}^{n+1} \frac{2}{i}$$

$$(n+1) S(0) + \frac{n+1}{n} \sum_{i=1}^{n+1} \frac{2}{i}$$

Harmonic series

same order $(n+1)\,1 + \frac{n+1}{n} 2n \log(n)$

$$\in O(n \log(n))$$