

## 2/5 Quick Sort and Bound

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### Quick Sort

Invented by Tony Hoare

#### Method

choose element to partition the array  
(pivot)

In linear time move elements less than  
the pivot to front of array  
and elements larger than pivot to back

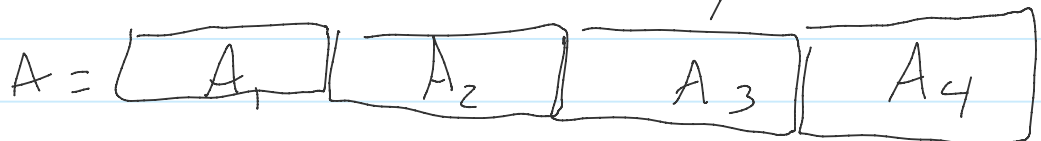
move pivot in between the two groups

Repeat recursively on front group  
and back group

Runtime on average case  $O(n \log n)$

#### 1) Percentile Proof

Let's consider the sorted array as  
the concatenation of 4 arrays



$\epsilon \cdot 1 \dots 1 \dots 1 \dots 1 \dots 1 \dots 1$

Each array has now equal length

The probability that a pivot chosen from  $A_2$  or  $A_3$  is roughly  $1/2$

If a pivot is chosen from  $A_2$  or  $A_3$

Then the runtime is in the worst case

$$T(n/4) + T(3n/4) + O(n)$$

Since Thus the maximum number of pivots chosen in the respective  $A_2 \cup A_3$  range becomes

$$\log_{4/3}(n)$$

The expected number of pivots chosen until an  $A_2 \cup A_3$  pivot is chosen is

$$1 \cdot (1/2) + 2 \cdot (1/2) \cdot (1-1/2) + 3 \cdot (1/2) \cdot (1-1/2)^2 + \dots$$

which can be expressed as

$$\sum_{i=1}^{\infty} i \cdot (1/2)^i$$

This is bounded by 2.

Thus the expected number of linear

Thus the expected number of linear search before a desirable pivot is chosen is  $\leq 2$

So the total expected run time becomes no worse than

$$2n \cdot \log_{4/3}(n) \in O(n \log n)$$

2) Recurrence Proof (or)  $E(T(i)) = S_i$

$$E(T(n)) = \frac{1}{n} (S(0) + S(n-1) + S(1) + S(n-2) + \dots + S(n-1) + S(0)) + (n-1)$$

Note each term  $S(i)$   $i \in [0, n-1]$  occurs twice

$$\rightarrow S(n) = \frac{2}{n} \sum_{i=0}^{n-1} S(i) + n - 1$$

$$S(n-1) = \frac{2}{n-1} \sum_{i=0}^{n-2} S(i) + n - 2$$

$$\left(\frac{n-1}{n}\right) \cdot S(n-1) = \frac{2}{n} \sum_{i=0}^{n-2} S(i) + \frac{(n-2)(n-1)}{n}$$

$$S(n) - \left(\frac{n-1}{n}\right) S(n-1) = \frac{2}{n} S(n-1) + \frac{(n-1)n}{n} - \frac{(n-2)(n-1)}{n}$$

$$S(n) = \frac{n+1}{n} S(n-1) + \frac{2}{n}$$

$$S(n-1) = \frac{n}{n-1} S(n-2) + \frac{2}{n-1}$$

$$S(n) = \frac{n+1}{n-1} S(n-2) + \frac{2}{n} + \frac{2(n+1)}{(n)(n-1)}$$

$$S(n) = \frac{n+1}{n-2} S(n-3) + \frac{2}{n} + \frac{2(n+1)}{n(n-1)} + \frac{2(n+1)}{n(n-2)}$$

$$\leq \frac{n+1}{k} (S(k-1)) + \frac{(n+1)}{n} \sum_{i=k}^{n+1} \frac{2}{i}$$

$$(n+1)S(0) + \frac{n+1}{n} \sum_{i=1}^{n+1} \frac{2}{i}$$



↓ Harmonic series

same order  $(n+1) | + \frac{n+1}{n} 2n \log(n)$

$$\in O(n \log(n))$$

